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Is a Spatially Closed Universe Possible?

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The present speculation shows that a spatially closed geometry might be an alternative to Inflation scenario.

KEY WORDS: early universe; flatness problem; horizon problem.

1. INTRODUCTION

Science improves with the abandonment of a priori hypotheses and/or not enough well-established principles. During the past decades cosmology has been faced with the problem of the meaning of the cosmological constant Λ, which asks whether it acts as a free parameter of Einstein equations or it is a fundamental constant of Nature. As long as Λ was assumed to be zero, a spatially closed geometry was synonymous with “Big Crunch,” and this is the reason why it was not understood as a realistic model as that. Nowadays, observations account for a cosmological constant that acts against gravitational attraction, but a vanishing curvature is prefered because of Inflation (Abbott, 1986; Guth, 1981). However, it turns out that this scenario given in the 80s for describing the early period of the expansion can be questioned (Triay, 2002).

Herein, I review the observational and theoretical features on which the standard world model is based without a priori hypotheses on the curvature parameter. Hence, I submit arguments in favor of a spatially closed geometry, which could turn to be an alternative to Inflation scenario (Triay, 1997b).

2. STANDARD PICTURE

Similarly to any reasonable scientific approach for understanding a complexe problem, the symmetries of the standard picture as defined by RW metric were settled at hand in order to provide us with an answer to the cosmological problem. Surprisingly, such a simple picture became a great success when compared to

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observations. Indeed, by using a geometrical interpretation of the CBR isotropy together with the recession of galaxies, it has been proved (Souriau, 1974) that the space-time V_4 has a RW-metric

$$ds^2 = dt^2 - a^2(t)\, d\sigma^2 \tag{1}$$

where $a(t)$ is the (dimensionless) expansion parameter ($a = 1$ today) and $d\sigma^2$ is the metric of the commoving space V_3. This mathematical proof rehabilitates the standard picture despite the presence of inhomogeneities in the space distribution of galaxies, which are observed up to scales of the order of 100 Mpc (galaxies space distribution, cosmic velocity fields, . . .). Friedmann–Lemaître–Gamow (FLG) model assumes a dust distribution with noninteracting radiation as gravitational sources. Einstein equations provide us with the dynamics of the expansion

$$H(t) = \frac{\dot{a}}{a} = H_0 \sqrt{\lambda_0 - \frac{k_0}{a^2} + \frac{\Omega_0}{a^3} + \frac{\alpha_0}{a^4}} \tag{2}$$

where H_0, λ_0, k_0, Ω_0, and α_0 denote the present values of cosmological parameters (usually denoted by Ω_Λ, Ω_K, Ω_M, and Ω_γ respectively). Namely, one has

- the reduced cosmological constant $\lambda = \frac{1}{3}\Lambda H^{-2}$;
- the curvature parameter $k = K H^{-2}$, where K is the curvature scalar of the commoving space V_3;
- the density parameter $\Omega = \frac{8}{3}\pi G_\rho H^{-2}$, where ρ stands for the specific density of massive particles ρ (dark matter included) and G is the Newton's constant;
- the radiation parameter $\alpha = \frac{8}{45}\pi^3 G(kT)^4 \hbar^{-3} H^{-2}$, which accounts for the CBR photon, where $\hbar$ is the Planck constant.

These parameters verify the normalization condition

$$1 = \lambda - k + \Omega + \alpha \tag{3}$$

which can be interpreted as a scale-free formulation of Eq. (2).

2.1. Prior to Decoupling

According to working hypotheses, this model does not account for epochs prior to decoupling, when the electrons combined with ions to form neutral H-atom. Extrapolations down to earlier epochs without necessary cautions lead to inherent problems in the standard picture. For example, it has been argued (Guth, 1981)

> . . . the standard model requires the assumption of initial conditions which are very implausible for two reasons:
>
> - Causally disconnected regions are assumed to be nearly identical; in particular they are simultaneously at the same temperature (the *Horizon problem*).

- For a fixed initial temperature, the value of the Hubble constant must be fine tuned to extraordinary accuracy to produce a universe which is as flat as the one we observe (the *Flatness problem*).

It is generally believed that these problems can be solved by adding an inflation era at primordial epoch of the expansion. Such a scenario has been initially motivated by quantum field theory in support of the SU(5) model of grand unification (GUT), in order to overcome the monopole problem. Although such a theory, which sought to unify the strong and the electro-weak interactions, turned out to be unsuccessful (Amoldi *et al.*, 1991; Montonet *et al.*, 1994). Inflation is still understood as a necessary ingredient of the standard picture. Nevertheless, a serious weakness of this approach is the lack of theoretical foundations.

3. A SPATIALLY CLOSED UNIVERSE

Statistical investigations based on the cosmological Hubble diagram of Brightest Cluster Galaxies (Bigot *et al.*, 1988; Bigot and Triay, 1989) and on the space and luminosity distributions QSOs samples (Fliche *et al.*, 1982; Fliche and Souriau, 1979; Triay, 1989) suggested a spatially closed FLG model $k_0 \sim 0.3$ with an eternal expansion and a cosmological constant $\Lambda \sim 3h^2 10^{-56}$ cm^{-2}. Two decades later, the Hubble diagram of SN data confirmed such a result (Perlmutter *et al.*, 1999). Nowadays, while a vanishing curvature is preferred, CMB data (Benoit *et al.*, 2003; Netterfield *et al.*, 2002; Sievers *et al.*, 2002; Spergel *et al.*, 2003) still suggest a closed universe $k_0 \sim 0.03$; see also Uzan *et al.* (2003). With such results in mind one may ask whether theoretical basis can justify a positive curvature for the universe.

3.1. Horizon Problem

At first glance, the Horizon problem has a natural issue in a spatially closed universe since the extrapolation of FLG model down to the origin of time provides us with a single event (Triay, 1996, 1997b). It is clear that a more realistic model has to account for primordial physics to properly answer this question, but such a task is quite difficult in essence because of nontrivial physics (baryons genesis, . . .) and open questions in relativistic thermodynamics. However, the property of compactness for the universe provides us with an enormous advantage for setting this problem in a different way. Indeed, one can safely expect that the true chronology of the primordial expansion is different from that defined by the FLG model $H \sim H_0\sqrt{\alpha_0}a^{-2}$. The CBR blackbody spectrum suggests that dissipative processes were present at primordial era since such effects are responsible for the trend toward equilibrium (e.g., at the nucleosynthesis epoch the density of matter is typical of the mean solar density). Such effects decelerate the expansion of the primordial universe with respect to FLG model, which makes a larger particle horizon. Hence, any smoothing process becomes more efficient for homogenizing and isotropizing the whole universe. Moreover, regardless of the effect of dissipative processes on geometry, it has been shown that a chaotic dynamics of a general Bianchi IX model takes enough time to converge toward a spatially closed FRW model (de Oliveira *et al.*, 2002).

3.2. Flatness Problem

The stringent initial boundary conditions on the value of the Hubble constant to produce a universe as flat as one sees today suggest a vanishing curvature (Guth, 1981) and thus an inflationary era (Linde, 1994). Flatness does not apply to the scalar curvature of the commoving space $K = K_0 a^{-2}$ but to its dimensionless measure

$$k = \frac{k_0 H_0^2}{a^2 H^2} \tag{4}$$

It means that the primordial Hubble length is very small compared to the scale of the universe. Such a property ($k \sim k_0 \alpha_0^{-1} a^2$ when $a \to 0$) is inherent in the FLG model and can be understood as an initial value problem (Triay, 1997a) from the epoch forward the universe can be described by the FLG model.

Actually, the original Flatness problem can be found as part of Dicke Coincidences (Dicke, 1970; Dicke and Peebles, 1979), the anthropic problem of why the curvature parameter k is not orders of magnitude different from zero today. A way to avoid such a coincidence (if any) is that the present values of cosmological parameters are close to their final values (obtained when $a \to +\infty$). Because $\Lambda \neq 0$, such a requirement provides us with the only issue $\lambda_0 \sim 1$ and $k_0 \sim \Omega_0$, which corresponds to a spatially close universe with a vacuum-dominated expansion (Triay, 1997a).

3.3. Origin of Structures

The problem on the origin of large scale structures is solved in part by the observed CMB fluctuations, which stand for seeds within a model of structures formation. It is generally believed that they correspond to quantum fluctuations that have been expanded up to cosmological scales according to Einstein equations. However, such a scenario becomes not as clear cut as that when the gravitational and quantum action units are compared (Triay, 2002). Indeed, because $\Lambda \neq 0$ one can choose units to rescale Einstein equations

$$T_{\mu\nu} = -g_{\mu\nu} + R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} \tag{5}$$

which sets the gravitational units of time $1/\sqrt{\Lambda}$ and of mass $1/(8\pi G\sqrt{\Lambda})$. Within such gravitational units, the quantum action unit $\hbar \sim 10^{-120}$ shows clearly that Einstein equations are not adapted for describing quantum physics. In other words, such a very small value for Planck constant (which has to be compared to $\hbar = 1$ when quantum units are used instead) makes it difficult that gravitation acts on quantum scales and vice versa. On the other hand, standard physics has plenty of resources and one might ask whether other candidate scenarios are compatible with CMB data. It is interesting to mention that the curvature parameter k_0, which is mainly the one that is estimated from such investigations, acts on the statistical likelihood of results because the size of the sample depends on it (the larger the curvature the less the fair sample) (Triay, 1996).

4. DISCUSSION

In addition to problems discussed above, which are know to justify the Inflation scenario, the standard picture does not account for a global vanishing electric charge (within an accuracy of 10^{-40}, otherwise the electrostatic repulsion makes the gravitational attraction not perceivable). A possible issue to this problem has been to restore a Baryon symmetric cosmology (matter and antimatter distributed in two separated hemispheres provides us with the lowest annihilation gamma-ray background) (Fliche *et al.*, 1982; Triay, 1989) in a FLG universe with regard to observations (Desert and Schatzmam, 1986). A spatially closed world model is also favored by fundamental considerations, such as the interpretation of origin of inertia (determined by the distribution and by the currents of mass-energy in the universe, see Cinfolini and Wheeler, 1995).

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